

Ballistic reconstruction of an incident involving a firearm

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Abstract

The article describes methods for determining the dimensionless drag coefficient of the bullet under examination, performing exterior ballistics calculations and using cartographic methods to determine the terrain profile in which an incident took place, employing new methods used in the Firearms, Toolmark and Road Accident Examination Department of the Central Forensic Laboratory of the Police to determine the firing position and to visualise the incident. The reconstruction of the incident is the result of these steps.

Keywords: dimensionless drag coefficient of a bullet, exterior ballistics calculations, terrain profile.

Introduction

In forensic practice there is sometimes a need for a specific kind of reconstruction of an incident involving a firearm, namely finding the firing position or visualising the bullet trajectory in the terrain. To this end, data obtained during the examination of physical evidence is used, such as the bullet, the firearm or the victim's clothing, and from an analysis of the case file material submitted, with particular regard to the crime scene examination report, the photographic documentation, the scene sketch, the medical or post-mortem report and other materials.

If the distance between the firing position and the point of impact (where the bullet was found) is considerable – several hundred metres or more (depending on the firearm and ammunition used) – computer calculation is used to determine the firing position.

For such cases, a three-stage procedure has been developed and implemented for carrying out the reconstruction:

- determining the C_D coefficient of the bullet,
- calculating the bullet trajectory,
- visualising the bullet trajectory in the terrain.

Determining the C_D coefficient of the bullet

One of the elements essential for the calculations is knowledge of the law governing the motion of the bullet in air. Various calculation methods, drag functions and so on can be used, but today, in an era of universal access to computers, it seems that better results are obtained by

using, in the calculations for a given bullet, the dependence of its drag coefficient on its velocity relative to the speed of sound in air, described as $C_D = f(Ma)$. This dependence is characteristic of a specific bullet (including its shape, mass and surface) and expresses how this value depends on the bullet's velocity, whereas older calculation methods make do with a constant value of C_D or of another parameter characterising the resistance of the bullet relative to the medium. Only general relationships can be found in the literature, and for bullets from small-arms ammunition or projectiles for air weapons the data of interest are usually given in simplified form. It is therefore best to determine them oneself for the specific case and then use them in a computer program for exterior ballistics calculations and, where appropriate, visualisation of the scene.

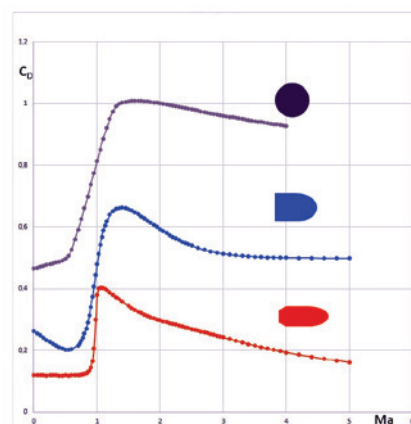


Fig. 1. General $C_D = f(Ma)$ relationships for various projectiles

Reputable ammunition manufacturers always state some value characterising the drag function of the bullets they produce. Most often this is the BC (ballistic coefficient) for the G1 or G7 drag function. This coefficient, however, is determined, for example, over a 100 m section of the trajectory, and thus for a bullet velocity close to the muzzle velocity. Meanwhile, the bullet's velocity, depending on the cartridge type from which it comes, decreases continuously and, after travelling generally 300–700 m of its trajectory, falls to a subsonic value. As shown in Figure 1, the changes in velocity are accompanied by a change in the drag coefficient C_D , i.e. a temporary increase followed by a significant decrease.

Attempts were made to determine the $C_D = f(\text{Ma})$ values in tabular form suitable for further use. A simple example was a 5.5 mm Gamo Expander pellet weighing 1 g fired from a Chinese-made 5.5 mm Snowpeak PP750 PCP air pistol (pre-charged pneumatic, i.e. with a built-in air reservoir charged, in this case, to about 250 bar).

Velocity was measured with a Canadian-made Labrador Doppler radar chronograph. The test shots were fired over a distance of 15 m, indoors and under stable temperature conditions.

The measurement file is saved in .csv format on the card inserted in the radar. The file was transferred to a computer and opened in a simple text editor, such as Windows Notepad, in order to replace the full stops with commas. The next steps require a spreadsheet program, such as Excel. A correctly imported sheet will consist of a header and four columns, as in Figure 2, where they are outlined with a red frame.

	A	B	C	D	E	F	G	H
1	Device ID	LBR-0029085						
2	Series No	1						
3	Shot No	4						
4								
5	Time (s)	Vel (m/s)	Dist (m)	SNR	Vlin [m/s]	A [m/s ²]	CD	Ma
6	0	174,4	0	-	174,91	-214,54		
7	0,035021	166,82	5,78	43,03	167,40	-199,66	0,500	0,490
8	0,037021	167,67	6,1	43,94	167,00	-198,05	0,498	0,489
9	0,039021	166,85	6,42	42,65	166,60	-196,44	0,496	0,488
10	0,041021	165,84	6,74	41,78	166,21	-194,83	0,495	0,486
11	0,043021	165,66	7,07	40,65	165,82	-193,22	0,493	0,485
12	0,045021	165,2	7,38	40,39	165,43	-191,61	0,491	0,484
13	0,047021	165,65	7,7	41,01	165,05	-190,01	0,489	0,483
14	0,049021	165,04	8,02	41,31	164,67	-188,40	0,487	0,482
15	0,051021	163,8	8,34	39,42	164,29	-186,79	0,485	0,481
16	0,053021	163,49	8,65	39,98	163,92	-185,18	0,483	0,480
17	0,055021	163,89	8,97	39,34	163,55	-183,57	0,481	0,479
18	0,057021	163,49	9,29	38,25	163,18	-181,96	0,479	0,478
19	0,059021	162,45	9,6	38,67	162,82	-180,35	0,477	0,476
20	0,061021	162,86	9,91	39,66	162,46	-178,75	0,475	0,475
21	0,063021	162,08	10,22	38,85	162,10	-177,14	0,473	0,474
22	0,065021	161,6	10,54	38,88	161,74	-175,53	0,471	0,473
23	0,067021	161,53	10,85	40,67	161,39	-173,92	0,468	0,472
24	0,069021	160,8	11,15	39	161,05	-172,31	0,466	0,471
25	0,071021	160,17	11,46	39,93	160,70	-170,70	0,464	0,470
26	0,073021	160,91	11,77	39,64	160,36	-169,09	0,461	0,469
27	0,075021	159,46	12,08	39,53	160,02	-167,49	0,459	0,468
28	0,077021	160,19	12,38	39,88	159,69			

Fig. 2. View of the spreadsheet columns – description in the text

Starting from row 6, the columns contain, respectively: A – time, B – velocity, C – distance and D – signal-to-noise ratio. The subsequent columns must be created by the user.

Column E – fitted (linearised) velocity – is created using the graph shown in Figure 3. The vertical axis

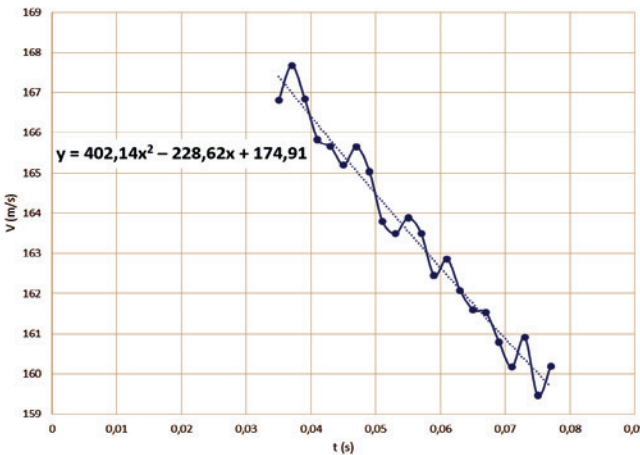


Fig. 3. View of the graph of velocity as a function of time – description in the text

here is column B – velocity – and the horizontal axis is column A – time; the values in row 6 are omitted, as they do not come from the measurement but are calculated by the radar software. A trendline is then added to the resulting graph using the spreadsheet's built-in functions, and when the option to display it is enabled, the trendline equation appears next to the graph. The trendline should preferably be a polynomial of degree no higher than 3. Over such short measurement sections, depending on the projectile, a straight-line equation or, as here, a parabola, i.e. a second-degree polynomial, works best. The trendline equation, i.e. the fitted velocity, is used to create column E – the time values taken from column A were entered as the x values. Next, the projectile deceleration values were calculated and placed in column F. In accordance with the definition, the change in velocity over time was sought, i.e. the result of the difference quotient $E7-E6/A7-A6$ was entered in cell F6, and analogously in the subsequent rows of column F. Next, the CD coefficient was calculated. For this purpose, the formula for the aerodynamic drag force known in aerodynamics and ballistics was used: $F = \frac{1}{2} C_D \rho S V^2$, where C_D is the dimensionless drag coefficient sought, ρ – air density, S – the cross-sectional area of the projectile and V^2 – the square of the projectile velocity. Newton's second law of motion was also used, substituting $F = m \cdot a$ (the sign is irrelevant here). After rearrangement, $C_D = ma / \frac{1}{2} \rho S V^2$ was obtained. The area of a circle, $\frac{1}{4} \pi d^2$, where d is the calibre of the projectile, was substituted for S , finally giving $C_D = 8ma / \rho V^2 \pi d^2$. Column G was created using this relationship, taking care to use the correct units, i.e. the projectile mass was entered in kilograms and its calibre in metres. Column H was created last; in it the Mach number was calculated by dividing the fitted velocity (from column E) by the speed of sound under the experimental conditions, taken as 343 m/s (or, more precisely, e.g. $Ma = V_{lin} / 20,56 \sqrt{T}$, where T is the ambient temperature in kelvins). The result of the calculations is columns G and H containing the dependence of CD on Ma. For the pellet examined, the coefficient obtained

varies only slightly, as the pellet moves within a narrow range of subsonic velocities. If the velocities of several pellets were measured and the calculations described above repeated several times, an averaged value of the coefficient would be obtained for use in further calculations.

The method described can also be used to determine the drag coefficient C_D of bullets moving over a wide range of velocities. For this purpose, preferably a dozen or more cartridges from a single lot should be prepared for testing. At least two of the cartridges should be disassembled to measure the diameter and mass of the bullet and the mass of the powder charge. The remaining cartridges should be divided into several test groups. In the first group the cartridges are left unchanged, as they will be used to determine the C_D coefficient for the muzzle velocity. Holes should be drilled in the case walls of the cartridges in the remaining groups, through which part of the powder charge will be poured out. In thin-walled cartridge cases, such as 5.6 mm "long rifle" rimfire cartridges, it is safer to file a small slit with a knife-edge file instead of drilling. Drilling is dangerous, as it may ignite the powder, rupture the case and eject the primer, as well as the bullet, at high velocity. A new, sharp drill bit should therefore be used, together with a fixture that holds the case securely. The fixture must have an open-work design to avoid a chamber effect, which in the event of ignition would cause a rise in pressure. The fixture should also be shielded, and personal protection for the face and the rest of the body should be used. The holes made in the case walls can be sealed with ordinary adhesive tape. From cartridges prepared in this way, powder can be poured out easily and safely into a container placed on the pan of a precision balance. The charges left should be such as to allow measurements over as much of the velocity range as possible. Given the variety of powder charges, it is impossible to give details here; in general, one can go down to about 20% of the charge mass. The radar data obtained from the measurements should be used for the calculations in the manner described above; the repetitive nature of some of the data makes it easier to "automate" their processing. The results obtained from shots at different velocities make it possible to "stitch together" the result in the form of the required $C_D = f(Ma)$ values in tabular form. When the powder charge is reduced, difficulties may be encountered with ignition, with the bullet lodging in the bore, or with unstable flight (tumbling) of the bullet.

Examples of drag functions determined in the manner described are given below.

Table 1. Drag function for an 8 g TFMJ bullet from a 9 mm Luger pistol cartridge with a Nontox primer, manufactured by S&B

Ma	0,10	0,30	0,60	0,80	0,90	0,95	1,00	1,10	1,20
C_D	0,20	0,21	0,23	0,25	0,32	0,50	0,80	0,70	0,65

Table 2. Drag function for an 8 g steel-core bullet from a Polish-made 7.62×39 mm intermediate cartridge

Ma	0,1	0,3	0,5	0,7	0,9	1,0	1,1	1,3	1,5	1,7	2,1
C_d	0,21	0,20	0,18	0,15	0,14	0,38	0,48	0,46	0,43	0,40	0,34

Calculating the bullet trajectory

The drag function determined is used for exterior ballistics calculations with the well-known EBV4 program. After starting the program and entering the bullet data, click Drag on the Data tab and then select Particular drag function – new file from the list. If old file is displayed instead of new file, change this by pressing the F5 function key, which acts as a toggle (Switch). A table will appear in which the drag function obtained from the experiments and calculations can be saved. After saving it under a distinctive name, select the View option to check that the function just saved is being used and that the Form factor equals 1. This is the value it should have, since we have just created and saved the drag function for our bullet, treated as the "reference" bullet.



Fig. 4. View of part of the EBV4 program

The remaining data are entered according to the circumstances, after which the shot is simulated and the result is saved to a file.

Visualising the bullet trajectory in the terrain

Any incident involving a firearm for which the firing position must be determined or the bullet trajectory visualised takes place in a particular terrain. The final stage of the reconstruction of such an incident involves preparing a sketch containing that terrain profile with a plot of the bullet trajectory determined above superimposed on it. The terrain profile can be obtained in various ways. This can be done independently at the scene, using measurements from a drone equipped with lidar. Measurements processed in appropriate software allow a complete elevation map to be recorded, from which the required profile can be derived. This yields

reliable and up-to-date data. Unfortunately, the cost of the equipment and software effectively hinders the development of this method of data acquisition.

What remains, therefore, is to use free software for viewing existing maps, editing and analysing spatial data and creating maps, such as QGIS. The program is easy to download and install in the usual way. Explaining how it works is beyond the scope of this article, so it is necessary to watch tutorial videos before proceeding. At the outset, attention should be paid to how to use the Browser panel and how to install plugins. In general, one needs to master the program sufficiently to create connections to WMS/WMTS and WCS services on a server, e.g. a geoportal, and to install the "Terrain profile" plugin used to record the necessary data. The place where the incident occurred is located on the displayed map. A line is then drawn connecting the firing position (if known), or the direction from which the bullet came, with the point of impact. Using the plugin, the data for the marked section can be read as a set of points with given distance and elevation, and then saved.

Finally, the incident is visualised by entering the saved profiles (both are sets of x and y points) into a spreadsheet, where they are easiest to present. Usually, a single simple chart is prepared, e.g. a scatter chart with smooth lines, containing two data series.

Summary

Several visualisations have already been carried out in the manner described. One of the first was the reconstruction of an incident at a shooting range, shown in Figure 5. A bullet fired at too high an angle flew over the baffles and struck and injured a bystander, passing through the victim's body and travelling on. For the visualisation, a terrain profile obtained from lidar was provided, together with data on the firearm, the ammunition, the firing position and the point of impact. Comparing the two profiles showed that the incident could have occurred under the prevailing conditions.

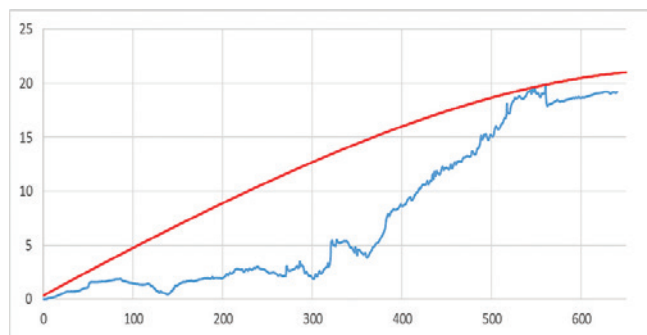


Fig. 5. Visualisation of the incident at the shooting range – description in the text

The second case was a very approximate visualisation of a person being shot with a hunting rifle bullet during a hunt, from a long distance. Only the calibre and type of the cartridge from which the bullet came, the place where the person was shot and the direction from which the bullet came were

regarded as certain data. Taking into account the very slight deformation of the bullet found, a considerable shooting distance was assumed – such that the bullet had a velocity of about 300 m/s on impact. The terrain conditions of the area in which the incident occurred, imported from a map service, made it possible to determine the possible trajectory shown in Figure 6.

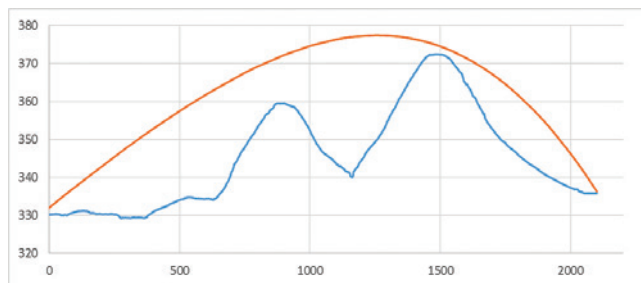


Fig. 6. Visualisation of the hunting incident. Description in the text

In conclusion, it should be stressed that a visualisation requires a great deal of data, usually going beyond the content of a typical crime scene examination. While individual elements of the reconstruction have been presented at meetings of police firearms and ballistics experts, they have never before been presented in such a comprehensive, though probably still insufficient, scope.

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